EC 623
ADVANCED DIGITAL SIGNAL PROCESSING

TERM-PROJECT

APPLICATION OF FILTER BANK THEORY TO SUBBAND CODING OF IMAGES

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Abstract
This project has been implemented as a part of EC 623 course- Advanced Digital Signal Processing, under the supervision of Dr. SRM Prasanna. The aim of this project is to study the basic principles of sub-band coding using the filter bank approach and its application in decomposition and coding of images. Also we wish to demonstrate the concept of sub-band coding of images by making use of some standard sub-band coding filters.

1. INTRODUCTION

One of the popular schemes for decomposition and coding of speech and image signals is Transform coding technique. Transform coding techniques decompose the source output into different frequency bands using block transforms. The transform coefficients have different characteristics and differing perceptual importance. These differences are exploited in allocating bits for encoding the different coefficients. This variable bit allocation results in a decrease in the average number of bits required to encode the source output. One of the drawbacks of transform coding is the artificial division of the source output into blocks, which results in the generation of coding artifacts at the block edges, or blocking. A popular approach to decomposing the image into different frequency bands without the imposition of an arbitrary block structure is sub-band coding. After the input has been decomposed into its constituents, we can use the coding technique best suited to each constituent to improve compression performance. Furthermore, each component of the source output may have different perceptual characteristics. Quantization error that is perceptually objectionable in one component may be acceptable in a different component of the source output. Therefore, a coarser quantizer may be used for perceptually less important components. This is how the concept of sub-band coding comes into picture.

2. SUB-BAND CODING ALGORITHM

The basic sub-band coding scheme is shown in Fig. 1.

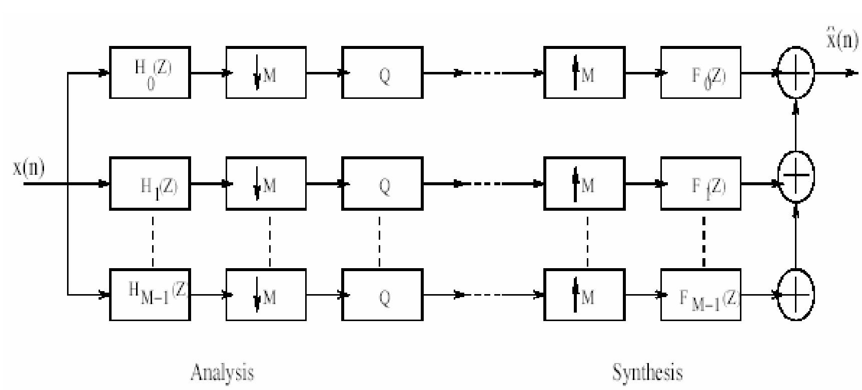

Fig. 1 Block diagram of sub-band coding system

2.1 ANALYSIS

The source output is passed through a bank of filters, called the analysis filter bank, which covers the range of frequencies that make up the source output. The passbands of the filter can be non-overlapping or overlapping. Non-overlapping and overlapping filter banks are shown in Fig. 2. The outputs of the filters are then sub-sampled.

The justification for the sub-sampling is the Nyquist Rule and its generalization, which tells us that we need only twice as many samples per second as the range of frequencies. This means that we can reduce the number of samples at the output of the filter because the range of frequencies at the output of the filter is less than the range of frequencies at the input to the filter. This process of reducing the number of samples is called *decimation,* or *down-sampling.* The amount of decimation depends on the ratio of the band-width of the filter output to the filter input. If the band width at the output of the filter is 1/M of the band-width at the input to the filter, we would decimate the output by a factor of M by keeping every Mth sample.
Once the output of the filters has been decimated, the output is encoded using one of the several schemes, including ADPCM, PCM, and vector quantization.

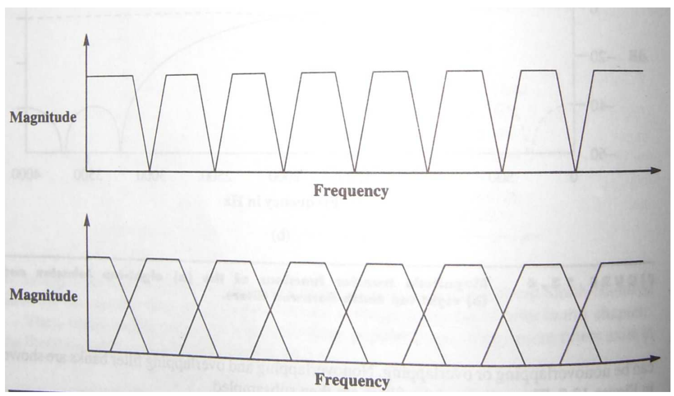

Fig. 2 Non-overlapping and overlapping filter banks

2.2 QUANTIZATION & CODING

Along with the selection of the compression scheme, the allocation of bits between the sub-bands is an important design parameter. Different sub-bands contain different amounts of information. Therefore, we need to allocate the available bits among the sub-bands according to some measure of the information content. The bit allocation procedure can have significant impact on the quality of reconstruction, especially when the information content of different bands is very different. If we use the variance of the output of each filter as a measure of information, and assume that the compression scheme is scalar quantization, we can arrive at several simple bit allocation schemes. One of the frequently used schemes for quantization and coding of transform coefficients is described below.

If the amount of information conveyed by each coefficient is different, it makes sense to assign differing number of bits to the different coefficients. There are two approaches to assigning bits. One approach relies on the average properties of the transform coefficients, while the other approach assigns bits as needed by individual transform coefficients.

In the first approach, we first obtain an estimate of the variances of the transform coefficients. These estimates can be used by one of the two algorithms to assign the number of bits used to quantize each of the coefficients. We assume that the relative variance of the coefficients corresponds to the amount of information contained in each coefficient. Thus, coefficients with higher variance are assigned more bits than coefficients with smaller variance.

We use the bit allocation that minimizes the distortion caused by quantization and encoding. If the average number of bits to be used per sample by the transform coding is R, and the average number of bits per sample used by the k^{th} coefficient is R_k, then

$$R = \frac{1}{M}\sum_{k=1}^{M} R_k \tag{1}$$

where, M is the number of transform coefficients. The reconstruction error variance for the kth quantizer $\sigma_{r_k}^2$ is related to the kth quantizer input variance $\sigma_{\theta_k}^2$ by the following:

$$\sigma_{r_k}^2 = \alpha_k 2^{-2R_k}\sigma_{\theta_k}^2 \tag{2}$$

where, α_k is a factor that depends on the input distribution and the quantizer.

The total reconstruction error is given by

$$\sigma_r^2 = \sum_{k=1}^{M} \alpha_k 2^{-2R_k}\sigma_{\theta_k}^2. \tag{3}$$

The objective of the bit allocation procedure is to find R_k to minimize (3) subject to the constraint of (1). The following expression for R_k, minimizes (3),

$$R_k = R + \frac{1}{2}\log_2 \frac{\sigma_{\theta_k}^2}{\prod_{k=1}^{M}(\sigma_{\theta_k}^2)^{\frac{1}{M}}}. \tag{4}$$

Although these values of R_k minimize (3), they are not guaranteed to be integers, or even positive. The standard approach at this point is to set the negative R_k's to zero. This will increase the average bit rate above the constraint. Therefore, the non-zero R_k's are uniformly reduced until the average rate is equal to R.

The second algorithm that uses estimates of the variance is a recursive algorithm and functions as follows:

1. Compute $\sigma_{\theta_k}^2$ for each coefficient.
2. Set R_k=0 for all k and set R_b=MR, where R_b is the total number of bits available for distribution.
3. Sort the variances {$\sigma_{\theta_k}^2$}. Suppose $\sigma_{\theta_l}^2$ is the maximum.
4. Increment R_l by 1, and divide $\sigma_{\theta_l}^2$ by 2.
5. Decrement R_b by 1. If R_b =0, then stop; otherwise go to 3.

If we follow this procedure, we end up allocating more bits to the coefficients with higher variance. This form of bit allocation is called zonal sampling. The advantage to this approach is its simplicity. Once the bit allocation has been obtained, every coefficient at a particular location is always quantized using the same number of bits. The disadvantage is that, because the bit allocations are performed based on average value, variations that occur on the local level are not reconstructed properly.

This problem can be avoided by using a different approach to bit allocation known as threshold coding. In this approach, which coefficient to keep, and which to discard, is not decided a priori. In the simplest form of threshold coding, we specify a threshold value. Coefficients with magnitude below this threshold are discarded, while the remaining are quantized and transmitted. The information about which coefficients have been retained is sent to the receiver as side information. An efficient approach is to scan the block of transformed coefficients in a zig-zag fashion. If we scan the block in this fashion, a large section of the tail-end of the scan will consist of zeros.

2.3 SYNTHESIS

The quantized and coded coefficients are used to reconstruct a representation of the original signal at the decoder. First, the encoded samples from each sub-band are decoded at the receiver. These decoded values are then upsampled by inserting an appropriate number of 0s between the samples. Once the number of samples per second is brought back to the original rate, the upsampled signals are passed through a bank of reconstruction filters. The outputs of the reconstruction filters are added to give the final reconstructed output.

Thus, the three major components of sub-band system are analysis and synthesis filters, the bit allocation scheme, and the encoding scheme.

3. SOME FILTERS USED IN SUB-BAND CODING

The most frequently used filters in sub-band coding consist of a cascade of stages, where each stage consists of a low-pass filter and a high-pass filter. The most popular among these are quadrature mirror filters (QMF). These filters have the property that if the impulse response of the low-pass filter is given by $\{h_n\}$, then the high-pass impulse response is given by $\{(-1)^n h_{N-1-n}\}$. The QMF filters designed by Johnston are widely used in a number of applications. The filter coefficients for 8-, and 16- tap filters are given in Tables 1 and 2. These filters are symmetric:

$$h_n = h_{N-1-n} \qquad n = 0,1,2,\ldots,N/2 -1. \qquad (5)$$

The filters with fewer taps are less efficient in their decomposition than the filters with more taps. However, the number of taps dictates the number of multiply-add operations necessary to generate the filter outputs. Thus, more efficient decomposition are obtained at the cost of increasing the computation.

Table 1 Coefficients for the 8-tap Johnston low-pass filter.

h0, h7	0.00938715
h1, h6	0.06942827
h2, h5	-0.07065183
h3, h4	0.48998080

Table 2 Coefficients for the 16-tap Johnston low-pass filter.

h0, h15	0.002898163
h1, h14	-0.009972252
h2, h13	-0.001920936
h3, h12	0.03596853
h4, h11	- 0.01611869
h5, h10	- 0.09530234
h6, h9	0.1067987
h7, h8	0.4773469

Another popular set of filters is the Smith-Barnwell filters, some of which are shown in Table 3. These families of filters differ in a number of ways. The magnitude transfer functions of 8-tap filters of these two families are plotted in Fig. 3. As can be seen, the cut-off for the Smith- Barnwell filter is much sharper than the cut-off for the Johnston filter. This means that the separation provided by the 8-tap Johnston filter is not as good as that provided by the 8-tap Smith-Barnwell filter.

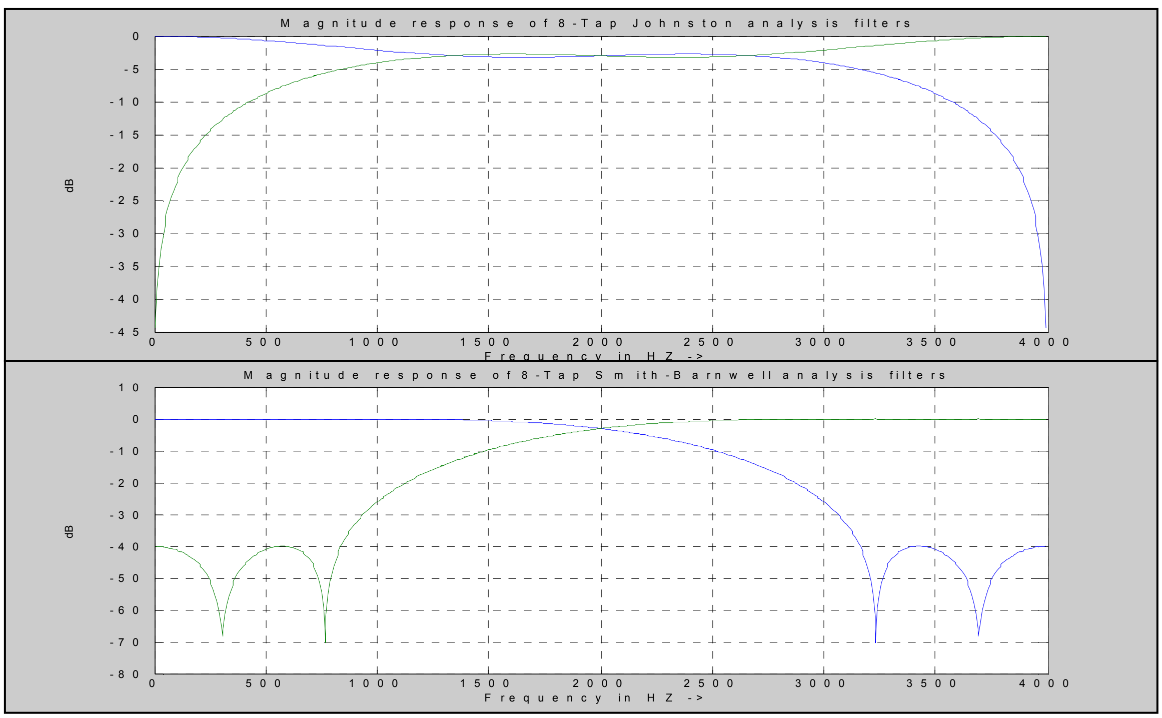

Fig. 3 Magnitude transfer functions of the (a) 8-tap Johnston, and (b) 8-tap Smith-Barnwell Filters.

3.1 M-BAND QMF FILTER BANKS

Various filter bank structures have been studied, in order to find filters that are simple to implement and provide good separation between the frequency bands. This section discusses some of the techniques used in the design of filter banks.

Table 3 Coefficients of the 8-tap Smith-Barnwell low-pass filter.

h0	0.0348975582178515
h1	-0.01098301946252854
h2	-0.06286453934951963
h3	0.223907720892568
h4	0.556856993531445
h5	0.357976304997285
h6	-0.02390027056113145
h7	-0.07594096379188282

In many applications it is necessary to divide the input into multiple bands. We can do this by a recursive two-band splitting as shown in Fig. 4, or we can obtain banks of filters that directly split into multiple bands. Given that we have good filters that provide two-band splitting, recursive splitting is an efficient way to design M-band QMF. However, even when the spectral characteristics of two-band filters are quite good, when applied to the tree structure of Fig. 4, the spectral characteristic may not be very good. This is because as we proceed through the tree, there is significant overlap between the bands, and hence there will be significant amount of aliasing in each band.

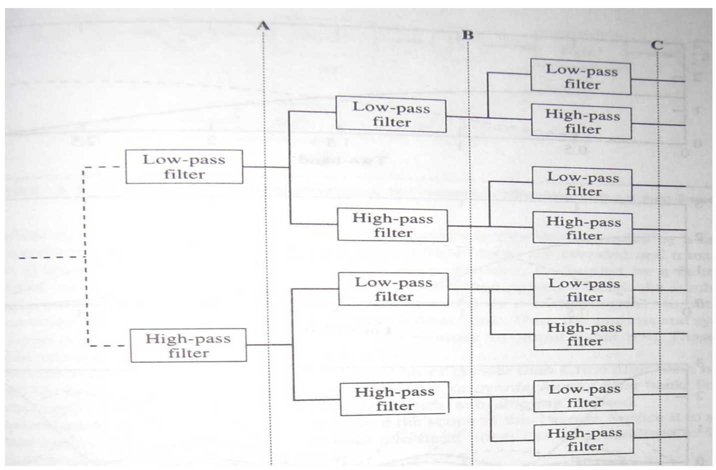

Fig. 4. Decomposition of an input sequence into multiple bands by recursively using a two-band split.

An M-band filter bank has two set of filters. The input sequence is split into M frequency bands using analysis bank of M-filters, and reconstructed by passing the sub-bands through synthesis bank of M-filters.

4. APPLICATION TO IMAGE COMPRESSION

So far, we have been concerned with sub-band coding of one-dimensional sequences. With two-dimensional sequences such as image, we need to use two-dimensional filters that separate the output into components based on both horizontal and vertical frequencies. Fortunately, in most cases, this two-dimensional filter can be implemented as two- one dimensional filters, which can be applied first in one-dimension, then in the other dimension. Filters that have this property are called separable filters.

Generally, for sub-band coding of images we filter each row of the image separately by using a high –pass and a low-pass filter. The output of the filter is decimated by a factor of two. Assume that the images were of size NxN. After the first stage, we have two NxN/2 images. We then process each image column-wise, and obtain four images of size N/2 x N/2. We can stop at this point or continue the decomposition process with one or more of the sub-images. Generally, of the four original sub-images, only one or two are further decomposed. The reason for not decomposing the other sub-images is that many of the pixel values in the high frequency sub-images are close to zero. Thus, there is little reason to spend computational power to decompose these sub-images.

4.1 DECOMPOSING AN IMAGE

Earlier a set of filters was provided to be used in one-dimensional sub-band coding. We can use those same filters to decompose an image into its sub-bands.

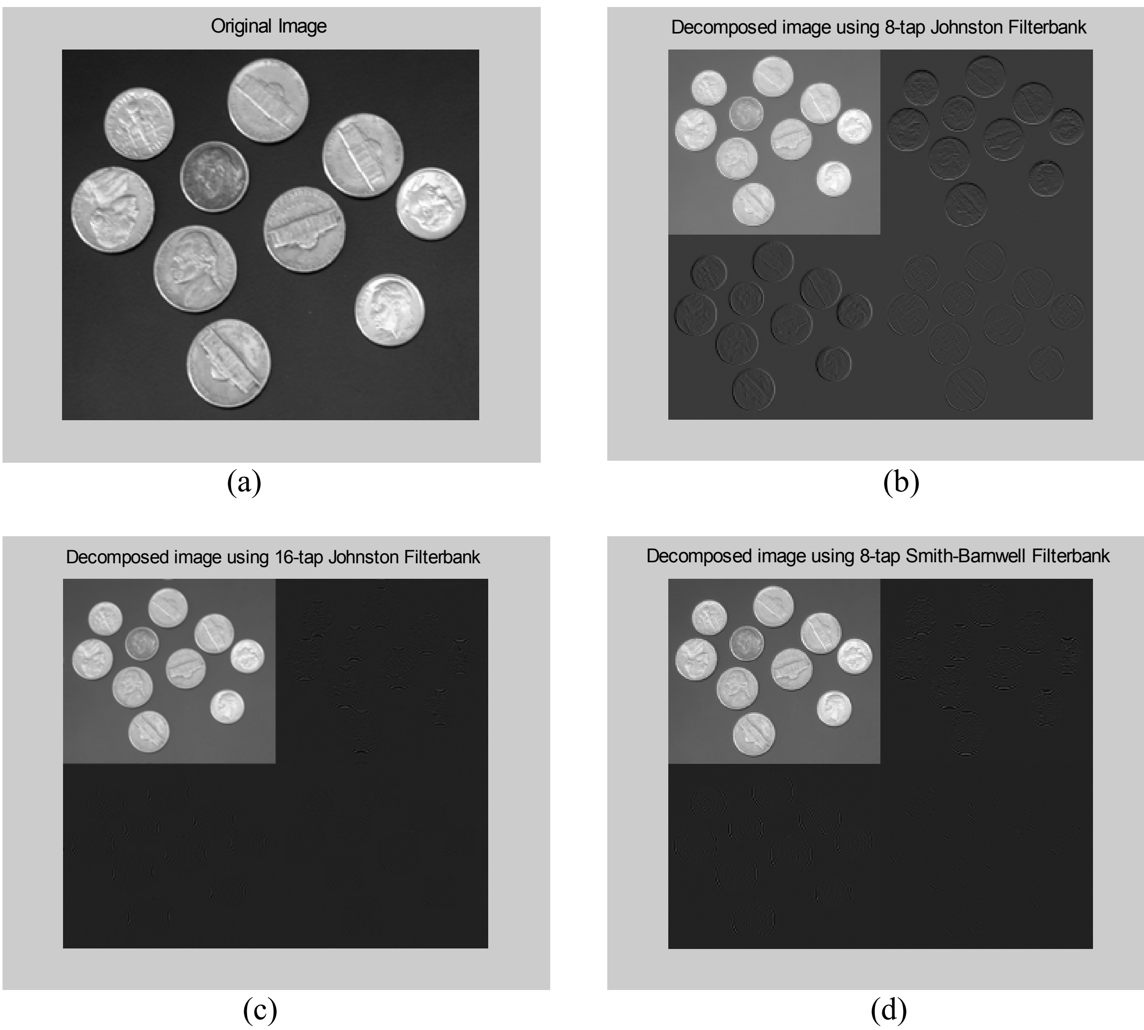

Fig. 5 Decomposition of (a) original image, using (b) Johnston 8-tap, (c) Johnston 16-tap, (d) Smith-Barnwell.

As can be seen, there is much less energy in the higher sub-bands. This difference in energy compaction can have drastic effect on the reconstruction. Fig. 5(c) shows the decomposition of the image using 16-tap Johnston Filter. As is evident, increasing the size of the filter improves the energy compaction. Fig. 5(d) illustrates decomposition of an image using 8-tap Smith-Barnwell filter. The results are almost identical to a 16-tap Johnston filter. Therefore, rather than increase the computational load by going to a 16-tap filter, we can keep the same computational load and use a different filter.

4.2 CODING THE SUB-BANDS

Once we have decomposed an image into sub-bands, we need to find the best encoding scheme for coding each sub-band. In this project, we use the coding scheme explained in 2.2. Fig. 6(b), and Fig. 6(d) compare the image coded at 0.5 bits per pixel using 8-tap Johnston filter and 8-tap Smith-Barnwell filter.

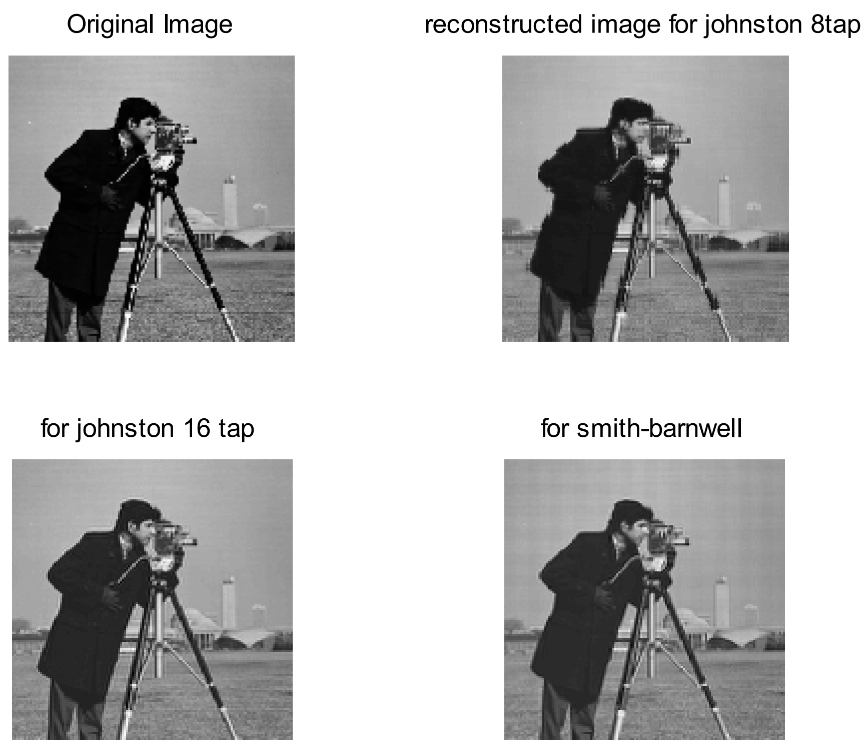

Fig. 6 (a) Original image, (b)Image coded at 0.5-bits per pixel using 8-tap Johnston filter, (c) 16-tap Johnston filter, (d) 8-tap Smith-Barnwell filter.

In the image decomposed using Johnston filter (Fig.5 (b)), there was significant energy in the high-low band. The algorithm allocated 1-bit to the low-low band and 1-bit to the high-low band. This resulted in poor encoding for both, and subsequently, poor reconstruction. There was very low energy in any of the bands other than the low-low band for the image decomposed using Smith-Barnwell filter. Therefore, the bit allocation algorithm assigned both bits to the low-low band which provided a reasonable reconstruction.

The issue of energy compaction becomes a very important factor in the reconstruction quality. Filters that allow for more energy compaction permit the allocation of bits to a smaller number of sub-bands. This in turn results in a better reconstruction.

5. CONCLUSION

Sub-band coding is any form of transform coding that breaks a signal into a number of different frequency bands and encodes each one independently. It is an important extension of Filter-bank theory and widely used in data compression.

6. REFERENCES

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